

WHAT INVARIANT ONE-PARTICLE MULTIPLICITY DISTRIBUTIONS AND TWO-PARTICLE CORRELATIONS ARE TELLING US ABOUT RELATIVISTIC HEAVY-ION COLLISIONS

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INTRODUCTION

Many of you are vigorously searching for the quark-gluon plasma—a predicted new phase of nuclear matter where quarks roam almost freely throughout the medium instead of being confined to individual nucleons.^{1,2} Such a plasma is believed to have existed in the first 10 μs of the universe during the big bang and could be produced in the laboratory during the little bang of a relativistic heavy-ion collision.

When nuclei collide head-on at relativistic speeds, the nuclear matter is initially compressed and excited from normal nuclear density and zero temperature to some maximum values—during which pions, kaons, and other particles are produced—and then expands, with a decrease in density and temperature. The early stages of the process are often treated in terms of nuclear fluid dynamics, but at some late stage the expanding matter freezes out into a collection of noninteracting hadrons.

To sample the density, temperature, collective velocity, size, and other properties of the system during this freeze-out, some of you are measuring invariant one-particle multiplicity distributions and two-particle correlations for the pions, kaons, and other particles that are produced. Your hope is that a sharp discontinuity in the value of one or more of the extracted freeze-out properties as a function of bombarding energy and/or size of the colliding nuclei could signal the formation of a quark-gluon plasma or other new physics. For the extraction of these freeze-out properties from your experimental measurements, a nine-parameter expanding source model was presented at the 12th Winter Workshop on Nuclear Dynamics.³

NINE-PARAMETER EXPANDING SOURCE MODEL

This source model describes invariant one-particle multiplicity distributions and two-particle correlations in nearly central relativistic heavy-ion collisions in terms of nine parameters, which are necessary and sufficient to characterize the gross properties of the source during its freeze-out from a nuclear fluid into a collection of noninteracting, free-streaming hadrons.^{3–5} The values of these nine parameters, along with their uncertainties at 99% confidence limits, are determined by minimizing χ^2 for the types of data considered. Several additional physically relevant quantities, along with their uncertainties at 99% confidence limits, can then be directly calculated. The nine independent source freeze-out properties that we consider here are the central baryon density n , nuclear temperature T , transverse collective velocity v_t , longitudinal collective velocity v_ℓ , source velocity v_s , transverse radius R_t , longitudinal proper time τ_f , width in proper time $\Delta\tau$, and pion incoherence fraction λ_π .

For a particular type of particle, the invariant one-particle multiplicity distribution and two-particle correlation function are calculated in terms of a Wigner distribution function $S(x, p)$, which is the phase-space density on the freeze-out hypersurface, giving the probability of producing a particle at spacetime point x with four-momentum p . It includes both a direct term⁶ and a term corresponding to 10 resonance decays.⁷ We consider nearly central collisions, assume axial symmetry, and work in cylindrical coordinates in the source frame, with longitudinal distance denoted by z , transverse distance denoted by ρ , and time denoted by t . Throughout the paper we use units in which $\hbar = c = k = 1$, where \hbar is Planck's constant divided by 2π , c is the speed of light, and k is the Boltzmann constant. However, for clarity, we reinsert c in the units of quantities whose values are given in the text or table.

Integration of the direct term over spacetime leads to the Cooper-Frye formula for the direct contribution to the invariant one-particle multiplicity distribution,⁸ namely

$$P_{\text{dir}}(p) = E \frac{d^3N_{\text{dir}}}{dp^3} = \frac{1}{2\pi m_t} \frac{d^2N_{\text{dir}}}{dy dm_t} = \frac{2J+1}{(2\pi)^3} \int_{\Sigma} d^3\sigma_{\mu} \frac{p^{\mu}}{\exp\{[p \cdot v(x) - \mu(x)]/T(x)\} \mp 1}, \quad (1)$$

where E denotes the particle's energy, $m_t = \sqrt{m^2 + p_t^2}$ its transverse mass, and y its rapidity. The quantity m is the particle's rest mass, and $p_t = \sqrt{p_x^2 + p_y^2}$ is its transverse momentum. The minus sign applies to bosons and the plus sign to fermions. The quantity J is the spin of the particle, $v(x)$ is the collective four-velocity, $T(x)$ is the nuclear temperature, and $\mu(x)$ is the chemical potential for this type of particle. We assume that the source is boost invariant within the limited region between its two ends,^{9,10} and that it starts expanding from an infinitesimally thin disk at time $t = 0$. The transverse velocity at any point on the freeze-out hypersurface, whose integration limits are denoted by Σ , is assumed to be linear in the transverse coordinate ρ .

For a particular type of particle, the two-particle correlation function is given by^{4,5,11}

$$C(K, q) = \frac{P_2(p_1, p_2)}{P(p_1) P(p_2)} = 1 \pm \lambda \frac{|\int d^4x S(x, K) \exp(iq \cdot x)|^2}{[\int d^4x S(x, p_1)][\int d^4x S(x, p_2)]}, \quad (2)$$

where $K = \frac{1}{2}(p_1 + p_2)$ is one-half the pair four-momentum and $q = p_1 - p_2$ is the pair four-momentum difference. In this equation the plus sign applies to bosons and the minus sign to fermions, and the quantity λ specifies the fraction of particles of this type that are produced incoherently.

APPLICATION TO Pb + Pb COLLISIONS AT $p_{\text{lab}}/A = 158$ GeV/c

We have used the nine-parameter expanding source model described in the previous section to analyze normalized but still preliminary data from Experiment NA44 performed at CERN's Super Proton Synchrotron.^{12,13} These data consist of invariant π^+ , K^+ , and K^- one-particle multiplicity distributions and π^+ and π^- two-particle correlations for the central 20% collisions in the reaction Pb + Pb at $p_{\text{lab}}/A = 158$ GeV/c.

For symmetric collisions the source velocity v_s can be calculated in terms of the beam momentum per nucleon and nucleon mass, which eliminates the need to vary this parameter. The remaining eight adjustable parameters are determined by minimizing χ^2 with a total of 2137 data points for the five types of data considered, so the number of degrees of freedom ν is 2129. The error for each point is calculated as the square root of the sum of the squares of its statistical error and its systematic error, with a systematic error of 15% for the π^+ , K^+ , and K^- one-particle multiplicity distributions and zero for the π^+ and π^- two-particle correlations. The resulting value of χ^2 is 2165.5, which corresponds to an acceptable value of $\chi^2/\nu = 1.017$. There is a 28.6% probability that χ^2/ν would be at least this large for a perfect model. The individual values of χ^2/ν are 0.904, 0.925, and 0.721 for the π^+ , K^+ , and K^- one-particle multiplicity distributions and are 1.130 and 1.012 for the π^+ and π^- two-particle correlations, respectively.

The values of the independent freeze-out properties determined this way, along with their uncertainties at 99% confidence limits on all quantities considered jointly, are given in the third column of Table 1. For comparison, we show in the second column of Table 1 the corresponding results⁴ for the central 7% collisions in the reaction Si + Au at $p_{\text{lab}}/A = 14.6$ GeV/c studied^{14,15} in Experiment E-802 at Brookhaven's Alternating Gradient Synchrotron. Compared to these earlier results for Si + Au collisions, the present results indicate that in Pb + Pb collisions the freeze-out density is somewhat lower, the freeze-out temperature is slightly higher, the source at freeze-out is somewhat larger, and the longitudinal collective velocity is very poorly determined (because of the limited experimental coverage in rapidity). The quantity n_0 appearing in Table 1 denotes normal nuclear density, whose value¹⁶ is calculated from the nuclear radius constant r_0 by means of $n_0 = 3/(4\pi r_0^3) = 3/[4\pi(1.16 \text{ fm})^3] = 0.153 \text{ fm}^{-3}$.

Table 1. Comparison of nine independent source freeze-out properties for the central 20% collisions of Pb + Pb at $p_{\text{lab}}/A = 158$ GeV/c with those for the central 7% collisions of Si + Au at $p_{\text{lab}}/A = 14.6$ GeV/c.

Property	Value and uncertainty at 99% confidence	
	Si + Au	Pb + Pb
Central baryon density n/n_0	$0.145^{+0.063}_{-0.045}$	$0.062^{+0.019}_{-0.015}$
Nuclear temperature T (MeV)	92.9 ± 4.4	95.8 ± 3.5
Transverse collective velocity v_t (c)	0.683 ± 0.048	0.664 ± 0.035
Longitudinal collective velocity v_ℓ (c)	$0.900^{+0.023}_{-0.029}$	$0.9985^{+0.0015}_{-0.94}$
Source velocity v_s (c)	$0.875^{+0.015}_{-0.016}$	0.9941219 (fixed)
Transverse radius R_t (fm)	8.0 ± 1.6	11.4 ± 1.5
Longitudinal proper time τ_f (fm/c)	8.2 ± 2.2	12.2 ± 2.1
Width in proper time $\Delta\tau$ (fm/c)	$5.9^{+4.4}_{-2.6}$	$7.1^{+4.5}_{-2.4}$
Pion incoherence fraction λ_π	0.65 ± 0.11	0.690 ± 0.074

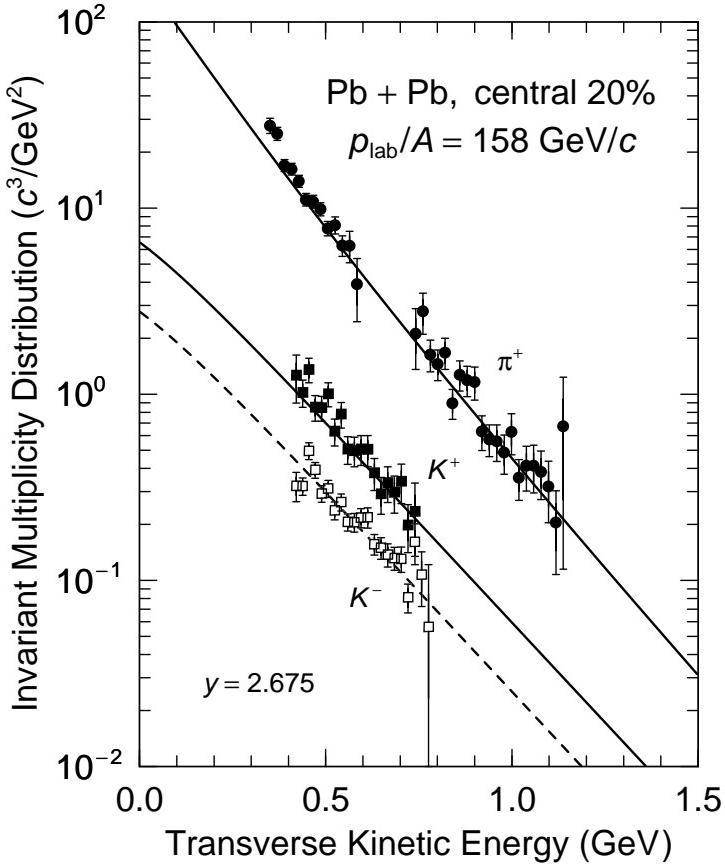


Figure 1. Comparison between model predictions and experimental data for the invariant one-particle multiplicity distribution $E d^3N/dp^3 = 1/(2\pi m_t) d^2N/dy dm_t$ for π^+ , K^+ , and K^- as a function of $m_t - m$ for $y = 2.675$. The error bars shown in this figure represent statistical errors only.

Figure 1 shows an example of our model predictions, given by the curves, compared with experimental data for the invariant one-particle multiplicity distribution for π^+ , K^+ , and K^- . Solid lines and solid symbols are used for positive particles, and a dashed line and open symbols are used for negative particles. The three curves are calculated for a value of rapidity $y = 2.675$, which is the central value of the kaon rapidity coverage in this experiment. The experimental data for kaons correspond to this same rapidity, with a rapidity bin width $\Delta y = 0.05$. For pions, the high- m_t experimental data (those to the right of the noticeable break in the distribution of points) correspond to this same rapidity and bin width, whereas the low- m_t experimental data correspond to the nearby rapidity $y = 2.65$ and rapidity bin width $\Delta y = 0.10$. The data shown in this figure represent only a small fraction of that used in our analysis.

Since the two-particle correlation function depends on five variables, it is somewhat difficult to graphically compare our model predictions with experimental data. The comparisons that we have made thus far involve fixing the pair rapidity and transverse momentum to specific values and then plotting the two-particle correlation function versus each component of the pair three-momentum difference, namely $q_{\text{longitudinal}}$, q_{side} , and q_{out} , for fixed values of the other two components. These comparisons demonstrate that our expanding model satisfactorily reproduces experimental two-particle correlations. However, as indicated earlier, the agreement between model predictions and experimental data is somewhat better for negative pions, where $\chi^2/\nu = 1.012$, than for positive pions, where $\chi^2/\nu = 1.130$.

RECONCILIATION WITH PREVIOUS ANALYSES

Analyses with our expanding source model for both the reaction Si + Au at $p_{\text{lab}}/A = 14.6 \text{ GeV}/c$ and the reaction Pb + Pb at $p_{\text{lab}}/A = 158 \text{ GeV}/c$ indicate that the freeze-out temperature is less than 100 MeV and that both the longitudinal and transverse collective velocities—which are anti-correlated with the temperature—are substantial. Similar conclusions concerning a low freeze-out temperature have also been reached by Csörgő and Lörstad.^{17,18} However, other analyses^{12,13,19–21} have yielded a much higher freeze-out temperature of approximately 140 MeV. In order to reconcile this serious discrepancy, we now examine the features in these analyses that led them to the conclusion of a much higher freeze-out temperature. These analyses fall into two major classes, which we consider in turn.

Neglect of Relativity in Extrapolation of Slope Parameters

One type of analysis^{12,13} was based upon the extrapolation to zero particle mass of extracted slope parameters characterizing the dependence of unnormalized transverse one-particle multiplicity distributions upon transverse mass. For a given reaction and type of particle, this transverse one-particle multiplicity distribution was represented by the expression*

$$\frac{1}{m_t} \frac{dN}{dm_t} = A \exp\left(-\frac{m_t}{T_{\text{eff}}}\right) , \quad (3)$$

where A is an arbitrary normalization constant and T_{eff} is the extracted slope parameter. Values of T_{eff} were extracted in this way for six types of particles originating from three separate reactions, namely π^+ , π^- , K^+ , K^- , p , and \bar{p} originating from the reaction $p + p$ at center-of-mass energy $\sqrt{s} = 23 \text{ GeV}$, from the central 10% collisions in the reaction S + S at $p_{\text{lab}}/A = 200 \text{ GeV}/c$, and from the central 6.4% collisions in the reaction Pb + Pb at $p_{\text{lab}}/A = 158 \text{ GeV}/c$.

As we will see below, the values of these extracted slope parameters contain valuable information, but they were unfortunately analyzed in Refs. 12 and 13 in terms of a heuristic equation that neglects relativity, namely

$$T_{\text{eff}} = T + m\bar{v}^2 , \quad (4)$$

where T is the nuclear temperature (whose value we are trying to determine) and \bar{v} is the average transverse collective velocity of the expanding matter from which the particle originated. On the basis of Eq. (4), the extrapolation in Ref. 13 of the extracted slope parameters to zero particle mass yielded the result $T \approx 140 \pm 15 \text{ MeV}$.

In the limit in which the particle velocity is large compared to the average collective velocity and with the aid of other simplifying assumptions and approximations, the correct relationship between slope parameter, nuclear temperature, particle mass, and average collective velocity can be easily derived from the relativistically correct Eq. (1). With the neglect of contributions from resonance decays, the neglect of the ∓ 1 appearing in the denominator of Eq. (1), the assumption of a constant freeze-out temperature, and the assumption that freeze-out occurs at a constant time t in the

*To facilitate comparisons with our own expressions, we have transformed the notation used in Refs. 12 and 13 to that used here.

source frame, Eq. (1) leads to

$$\frac{1}{m_t} \frac{d^2N}{dy dm_t} = A'E \int_V d^3x \exp\left[-\frac{p \cdot v(\mathbf{x})}{T}\right] = A'E \int_V d^3x \exp\left\{-\frac{\gamma(\mathbf{x})[E - \mathbf{p} \cdot \mathbf{v}(\mathbf{x})]}{T}\right\}, \quad (5)$$

where A' is a different arbitrary normalization constant from the one appearing in Eq. (3), the subscript V on the integral denotes the spatial integration limits for the source, and the position-dependent Lorentz factor $\gamma(\mathbf{x}) = 1/\sqrt{1 - \mathbf{v}(\mathbf{x}) \cdot \mathbf{v}(\mathbf{x})}$.

By introducing an average collective velocity \bar{v} in the integrations in Eq. (5), taking the limit in which the particle velocity is large compared to the collective velocity, specializing to the transverse direction, and neglecting the pre-exponential E dependence, we are led to

$$\frac{1}{m_t} \frac{dN}{dm_t} = A \exp\left[-\frac{\bar{\gamma}(m_t - p_t \bar{v})}{T}\right] = A \exp\left(-\frac{m_t - \bar{v}\sqrt{m_t^2 - m^2}}{T\sqrt{1 - \bar{v}^2}}\right). \quad (6)$$

To obtain the relationship between the slope parameter, nuclear temperature, particle mass, and average collective velocity, we equate the derivatives with respect to m_t of Eqs. (3) and (6), which leads to

$$T = \left(1 - \frac{\bar{v}m_t}{p_t}\right) \frac{T_{\text{eff}}}{\sqrt{1 - \bar{v}^2}} = \left(1 - \bar{v}\sqrt{1 + \frac{m^2}{p_t^2}}\right) \frac{T_{\text{eff}}}{\sqrt{1 - \bar{v}^2}}. \quad (7)$$

An analogous relationship has also been obtained by Siemens and Rasmussen²² for the case of a blast wave produced by the explosion of a spherically symmetric fireball.

In the limit of zero particle mass, Eq. (7) reduces to

$$T = T_{\text{eff}} \sqrt{\frac{1 - \bar{v}}{1 + \bar{v}}}, \quad (8)$$

which agrees with the result obtained by Schnedermann, Sollfrank, and Heinz^{23,24} for the case of cylindrical symmetry. With a typical value of $0.4c$ for the average collective velocity \bar{v} and the limiting value of $T_{\text{eff}} \approx 140 \pm 15$ MeV obtained in Ref. 13 by extrapolating slope parameters to zero particle mass, Eq. (8) yields $T \approx 92 \pm 10$ MeV for the nuclear temperature.

Several Approximations Made in a Thermal Model

Another type of analysis^{12,13,19–21} utilized the thermal model of Schnedermann, Sollfrank, and Heinz^{23,24} to extract the nuclear temperature and transverse surface collective velocity from unnormalized experimental transverse one-particle multiplicity distributions. An accumulation of effects from several approximations led to a somewhat higher temperature than we have found with our expanding source model. These approximations include the neglect of contributions from resonance decays, the neglect of the ∓ 1 appearing in the denominator of Eq. (1), the neglect of the coupling of the transverse motion to the longitudinal motion, and—most importantly—the neglect of information contained in the absolute normalization of the multiplicity distributions. The accumulation of effects from these approximations was responsible for the conclusion on page 2083 of Ref. 13 that “Within a temperature range $100 \leq T \leq 150$ MeV, the fits are equally good.” It is seen that the use of unnormalized experimental transverse one-particle multiplicity distributions in such a thermal model provides only a rough indication of the nuclear temperature at freeze-out.

ADDITIONAL STUDIES WITH EXPANDING SOURCE MODEL

To test the robustness of our expanding source model, we used it to analyze one-particle and correlation data generated theoretically from a nuclear fluid dynamical calculation performed with the computer program HYLANDER²⁵ that corresponded to a high freeze-out temperature and an extremely low transverse freeze-out velocity. The results of this study demonstrated that the model is capable of reproducing the underlying freeze-out properties even when their values were chosen to lie in unanticipated regions.

To determine whether or not ultrarelativistic proton-proton collisions can be described in terms of nuclear fluid dynamics, we used our expanding source model to analyze invariant π^+ , π^- , K^+ , K^- , p , and \bar{p} one-particle multiplicity distributions²⁶ for the reaction $p + p$ at center-of-mass energy $\sqrt{s} = 45$ GeV. We included the systematic and normalization errors discussed in Ref. 26 in addition to statistical errors. Because the pion incoherence fraction λ_π does not enter in the expression for one-particle multiplicity distributions and because for symmetric collisions the source velocity v_s can be calculated, there are only seven adjustable parameters in this case. These parameters are determined by minimizing χ^2 with a total of 459 data points for the six types of data considered, so the number of degrees of freedom ν is 452. The resulting value of χ^2 is 1132.0, which corresponds to a completely unacceptable value of $\chi^2/\nu = 2.504$. The probability that a perfect model would have resulted in a value of χ^2 at least as large as that found here is the incredibly small value 4.9×10^{-60} .

SUMMARY AND CONCLUSIONS

We have used a nine-parameter expanding source model that includes special relativity, quantum statistics, resonance decays, and freeze-out on a realistic hypersurface in spacetime to analyze in detail invariant π^+ , K^+ , and K^- one-particle multiplicity distributions and π^+ and π^- two-particle correlations in nearly central collisions of Pb + Pb at $p_{\text{lab}}/A = 158$ GeV/c. These studies confirm an earlier conclusion;^{3–5} for nearly central collisions of Si + Au at $p_{\text{lab}}/A = 14.6$ GeV/c the freeze-out temperature is less than 100 MeV and both the longitudinal and transverse collective velocities—which are anti-correlated with the temperature—are substantial.

We also reconciled our current results with those of previous analyses that yielded a much higher freeze-out temperature of approximately 140 MeV for both Pb + Pb collisions at $p_{\text{lab}}/A = 158$ GeV/c and other reactions. One type of analysis was based upon the use of a heuristic equation that neglects relativity to extrapolate slope parameters to zero particle mass. Another type of analysis utilized a thermal model in which there was an accumulation of effects from several approximations.

The future should witness the arrival of much new data on invariant one-particle multiplicity distributions and two-particle correlations as functions of bombarding energy and/or size of the colliding nuclei. The proper analysis of these data in terms of a realistic model could yield accurate values for the density, temperature, collective velocity, size, and other properties of the expanding matter as it freezes out into a collection of noninteracting hadrons. A sharp discontinuity in the value of one or more of these properties could conceivably be the long-awaited signal for the formation of a quark-gluon plasma or other new physics.

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